UK INTERMEDIATE MATHEMATICAL CHALLENGE

THURSDAY 4TH FEBRUARY 2010

Organised by the **United Kingdom Mathematics Trust**
and supported by

The Actuarial Profession
making financial sense of the future

RULES AND GUIDELINES (to be read before starting)

1. Do not open the paper until the Invigilator tells you to do so.

2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.

3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.

4. Candidates in England and Wales must be in School Year 11 or below.
   Candidates in Scotland must be in S4 or below.
   Candidates in Northern Ireland must be in School Year 12 or below.

5. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the
   Answer Sheet for each question. Do not mark more than one option.

6. *Do not expect to finish the whole paper in 1 hour.* Concentrate first on Questions 1-15.
   When you have checked your answers to these, have a go at some of the later questions.

7. Five marks are awarded for each correct answer to Questions 1-15.
   Six marks are awarded for each correct answer to Questions 16-25.
   Each incorrect answer to Questions 16-20 loses 1 mark.
   Each incorrect answer to Questions 21-25 loses 2 marks.

8. Your Answer Sheet will be read only by a *dumb machine*. **Do not write or doodle on the
   sheet except to mark your chosen options**. The machine 'sees' all black pencil markings
   even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of
   rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.

9. The questions on this paper challenge you to **think**, not to guess. You get more marks,
   and more satisfaction, by doing one question carefully than by guessing lots of answers.
   The UK IMC is about solving interesting problems, not about lucky guessing.

1. What is the value of $10 + 10 \times 10 \times (10 + 10)$?

   A 21 000   B 20 100   C 2100   D 2010   E 210

2. Three of the interior angles of a given quadrilateral are each 80°. What is the fourth angle of this quadrilateral?

   A 120°   B 110°   C 100°   D 90°   E 80°

3. Exactly one of the following is a prime number. Which is it?

   A 2345   B 23 456   C 234 567   D 2 345 678   E 23 456 789

4. A radio advertisement claimed that using a particular brand of artificial sweetener every day would 'save 7 000 calories in a year'. Approximately how many calories is this per day?

   A 20   B 40   C 70   D 100   E 140

5. Which of the following has the greatest value?

   A one half of $\dfrac{1}{25}$ B one third of $\dfrac{1}{20}$ C one quarter of $\dfrac{1}{15}$

   D one fifth of $\dfrac{1}{10}$ E one sixth of $\dfrac{1}{5}$

6. In triangle *PQR*, *S* is a point on *QR* such that $QS = SP = PR$ and $\angle QPS = 20°$. What is the size of $\angle PRS$?

   A 20°   B 35°   C 40°   D 55°   E 60°
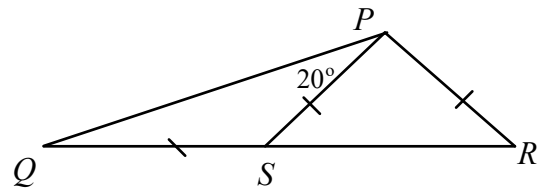
7. The Three Choirs Festival is held annually. Its venue rotates in a three-year cycle among Hereford, Gloucester and Worcester. In 2009, it was held in Hereford, in 2010 it will be held in Gloucester, next year it will be held in Worcester.

   Assuming that this three-year cycle continues, in which one of the following years will the Festival *not* be held in Worcester?

   A 2020   B 2032   C 2047   D 2054   E 2077

8. On my clock's display, the time has just changed to 02:31. How many minutes will it be until all the digits 0, 1, 2, 3 next appear together again?

   A 1   B 41   C 50   D 60   E 61

9. The perimeters of the three shapes shown are made up of straight lines and semi-circular arcs of diameter 2. They will fit snugly together as in a jigsaw.

   What is the difference between the total perimeter of the three separate pieces and the perimeter of the shape formed when the three pieces fit together?

   A 0   B $2 + 2\pi$ C $8 + 4\pi$ D $22 + 2\pi$ E $30 + 6\pi$
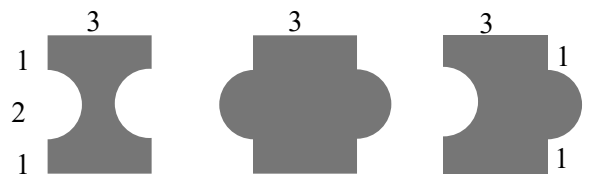
10. One year in the 1990s, January 1st fell on a Monday. Eleven years later, January 1st was also a Monday. How many times did February 29th occur during those eleven years?

    A 1   B 2   C 3   D 4   E 5

11. "You eat more than I do," said Tweedledee to Tweedledum.
    "That is not true," said Tweedledum to Tweedledee.
    "You are both wrong," said Alice, to them both.
    "You are right," said the White Rabbit to Alice.
    How many of the four statements were true?

   A 0   B 1   C 2   D 3   E 4

12. A cuboid is cut away from a cube of side 10 cm as shown.
    By what fraction does the total surface area of the solid decrease as a result?

    A $\dfrac{1}{4}$ B $\dfrac{1}{6}$ C $\dfrac{1}{10}$ D $\dfrac{1}{12}$ E $\dfrac{1}{18}$
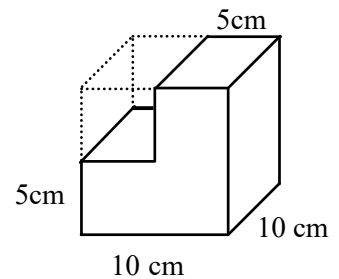

13. At Corbett's Ironmongery a fork handle and a candle cost a total of £6.10. The fork handle costs £4.60 more than the candle. What is the cost of two fork handles and four candles?

    A £14.45   B £13.70   C £12.95   D £12.20   E £8.35

14. Given that $4x - y = 5$, $4y - z = 7$ and $4z - x = 18$, what is the value of $x + y + z$?

    A 8   B 9   C 10   D 11   E 12

15. Bill is trying to sketch the graph of $y = 2x + 6$ but in drawing the axes he has placed the x-axis up the page and the y-axis across the page. Which of these five graphs is a correct sketch of $y = 2x + 6$ when the axes are placed in this way?
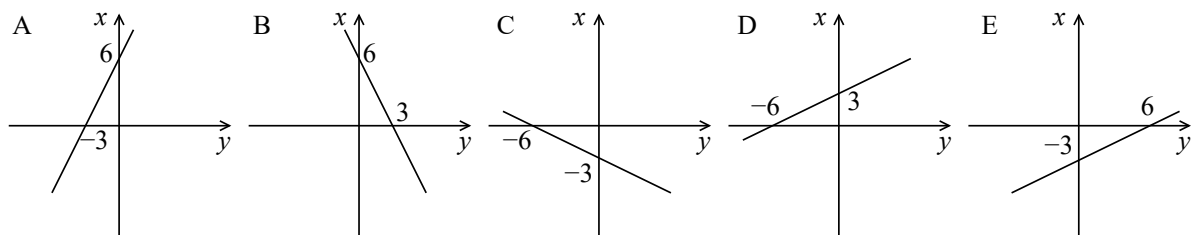

16. Albert Einstein is experimenting with two unusual clocks which both have 24-hour displays. One clock goes at twice the normal speed. The other clock goes backwards, but at the normal speed. Both clocks show the correct time at 13:00.
    What is the correct time when the displays on the clocks next agree?

    A 05:00   B 09:00   C 13:00   D 17:00   E 21:00

17. Last year Gill's cylindrical 21st birthday cake wasn't big enough to feed all her friends. This year she will double the radius and triple the height. What will be the ratio of the volume of this year's birthday cake to the volume of last year's cake?

    A 12:1   B 7:1   C 6:1   D 4:1   E 3:1

18. Supergran walks from her chalet to the top of the mountain. She knows that if she walks at a speed of 6 mph she will arrive at 1 pm, whereas if she leaves at the same time and walks at 10 mph, she will arrive at 11 am.
    At what speed should she walk if she wants to arrive at 12 noon?

    A 7.5 mph   B $7\frac{1}{7}$ mph   C 7.75 mph   D $\sqrt{60}$ mph   E 8 mph

19. A snail is at one corner of the top face of a cube with side length 1 m. The snail can crawl at a speed of 1 m per hour. What proportion of the cube's surface is made up of points which the snail could reach within one hour?

    A $\dfrac{\pi}{16}$ B $\dfrac{\pi}{8}$ C $\dfrac{1}{4}$ D $\dfrac{1}{2}$ E $\dfrac{\sqrt{3}}{4}$

20. Shahbaz thinks of an integer, n, such that the difference between $\sqrt{n}$ and 7 is less than 1.
    How many different possibilities are there for n?

    A 13   B 14   C 26   D 27   E 28

21. A square maze has 9 rooms with gaps in the walls between them. Once a person has travelled through a gap in the wall it then closes behind them. How many different ways can someone travel through the maze from X to Y?
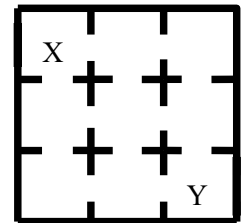
    A 8   B 10   C 12   D 14   E 16

22. Curly and Larry like to have their orange squash made to the same strength. Unfortunately, Moe has put 25 ml of squash with 175 ml of water in Curly's glass and 15 ml of squash with 185 ml of water in Larry's glass. How many millilitres of the mixture in Curly's glass must be put into Larry's glass so that they end up with drinks of the same strength?

    A 5   B 7   C 10   D 12   E it is not possible

23. The diagram shows a pattern of eight equal shaded squares inside a circle of area π square units. What is the area (in square units) of the shaded region?
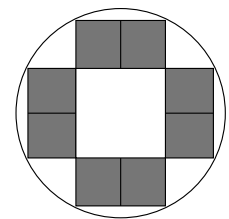
    A $1\frac{1}{3}$ B $1\frac{3}{5}$ C $1\frac{2}{3}$ D $1\frac{7}{9}$ E 2

24. A new taxi firm needs a memorable phone number. They want a number which has a maximum of two different digits. Their phone number must start with the digit 3 and be six digits long. How many such numbers are possible?

    A 288   B 280   C 279   D 226   E 225

25. Two squares, each of side length $1 + \sqrt{2}$ units, overlap. The overlapping region is a regular octagon.
    What is the area (in square units) of the octagon?

    A $1 + \sqrt{2}$ B $1 + 2\sqrt{2}$ C $2 + \sqrt{2}$ D $2 + 2\sqrt{2}$ E $2 + 3\sqrt{2}$